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SOME BOUNDS FOR ALTERNATING MATHIEU TYPE SERIES

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Abstract. Using recent investigated integral representations for the generalized alternating Mathieu series $\tilde{S}_{\mu}^{(\alpha,\beta)}\left(r;\{a_n\}_{n=1}^{\infty}\right)$ $\left(r,\alpha,\beta,\mu,\ \{a_n\}_{n=1}^{\infty}\in R^{+}\right)$ [9,14,18] with $a_n=n^{\gamma}$, $\gamma\in R^{+}$ and Mellin-Laplace type integral transforms for the generalized hypergeometric functions and the Bessel function of first kind, some bounding inequalities for $\tilde{S}_{\mu}^{(\alpha,\beta)}\left(r;\ \{n^{\gamma}\}_{n=1}^{\infty}\right)$ are presented. Namely, it is shown that the series $\tilde{S}_{\mu}^{(\alpha,\beta)}\left(r;\{n^{\gamma}\}_{n=1}^{\infty}\right)$ under some conditions for parameters α, β, γ and μ are bounded with constants which do not depend on α,β and γ but only depend on r and μ, i.e.

$$\tilde{S}_{\mu}^{(\alpha,\beta)}\left(r;\{n^{\gamma}\}_{n=1}^{\infty}\right)\leqslant\frac{2}{(1+r^2)^{\mu}}.$$

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