

CHARACTERIZATIONS OF CONVEX FUNCTIONS OF A VECTOR VARIABLE VIA HERMITE–HADAMARD’S INEQUALITY

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Abstract. The classical Hermite–Hadamard inequality characterizes the continuous convex functions of one real variable. The aim of the present paper is to give an analogous characterization for functions of a vector variable.

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