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MEANS AND HERMITE INTERPOLATION

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Abstract. Let $m_2 < m_1$ be two given nonnegative integers with $n = m_1 + m_2 + 1$. For suitably differentiable f, we let $P, Q \in \pi_n$ be the Hermite polynomial interpolants to f which satisfy $P^{(j)}(a) = f^{(j)}(a), j = 0, 1, ..., m_1$ and $P^{(j)}(b) = f^{(j)}(b), j = 0, 1, ..., m_2$, $Q^{(j)}(a) = f^{(j)}(a), j = 0, 1, ..., m_2$ and $Q^{(j)}(b) = f^{(j)}(b), j = 0, 1, ..., m_1$. Suppose that $f \in C^{n+2}(I)$ with $f^{(n+1)}(x) \neq 0$ for $x \in (a, b)$. If $m_1 - m_2$ is even, then there is a unique $x_0, a < x_0 < b$, such that $P(x_0) = Q(x_0)$. If $m_1 - m_2$ is odd, then there is a unique $x_0, a < x_0 < b$, such that $f(x_0) = \frac{1}{2}(P(x_0) + Q(x_0))$. x_0 defines a strict, symmetric mean, which we denote by $M_{f,m_1,m_2}(a, b)$. We prove various properties of these means. In particular, we show that $f(x) = x^{m_1+m_2+2}$ yields the arithmetic mean, $f(x) = x^{-1}$ yields the harmonic mean, and $f(x) = x^{(m_1+m_2+1)/2}$ yields the geometric mean.

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