

## DYNAMIC DOUBLE INTEGRAL INEQUALITIES IN TWO INDEPENDENT VARIABLES ON TIME SCALES

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*Abstract.* First, we establish some new nonlinear dynamic inequalities in two independent variables of Pachpatte type, that might be useful tools in the study of qualitative properties of solutions of certain classes of dynamic equations on time scales. These results extend recent inequalities for difference equations to the general time-scale setting. Then, after establishing a nabla Jensen's inequality, we relate several inequalities of Hilbert-Pachpatte type that extend and unify recent continuous and discrete inequalities of this type.

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