S–CONVEX EXTREMAL DISTRIBUTIONS
WITH ARBITRARY DISCRETE SUPPORT

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Abstract. This paper considers the class of s-convex stochastic orderings for random variables valued in an arbitrary discrete subset of the half-positive real line. After having established a sufficient condition of crossing-type for these orderings, explicit expressions are derived for stochastic extrema in moment spaces. Some applications in actuarial science are discussed.

REFERENCES

[1] C. COURTOIS, & M. DENUIT, *Convex bounds on multiplicative processes, with applications to pricing in incomplete markets*, Insurance: Mathematics and Economics, In Press (2007).

[2] C. COURTOIS, M. DENUIT, & S. VAN BELLEGEM, *Discrete s-convex extremal distributions: Theory and applications*, Applied Mathematics Letters **19**, 1367–1377 (2006).

[3] M. DENUIT, J. DHAENE, M. GOOVAERTS, & R. KAAS, *Actuarial Theory for Dependent Risks: Measures, Orders and Models*, Wiley, New York (2005).

[4] M. DENUIT, & CL. LEFÈVRE, *Some new classes of stochastic order relations among arithmetic random variables, with applications in actuarial sciences*, Insurance: Mathematics and Economics **20**, 197–213 (1997).

[5] M. DENUIT, CL. LEFÈVRE, & M. MESFIOUI, *On s-convex stochastic extrema for arithmetic risks*, Insurance: Mathematics and Economics **25**, 143–155 (1999).

[6] M. DENUIT, CL. LEFÈVRE, & M. SHAKED, *The s-convex orders among real random variables, with applications*, Mathematical Inequalities & Applications **1**, 585–613 (1998).

[7] M. DENUIT, CL. LEFÈVRE, & S. UTEV, *Stochastic orderings of (convex/concave)-type on an arbitrary grid*, Mathematics of Operations Research **24**, 835–846 (1999).

[8] P. C. FISHBURN, & I. H. LAVALLE, *Stochastic dominance on unidimensional grids*, Mathematics of Operations Research **20**, 513–525 (Erratum in Vol. 21 p. 252) (1995).

[9] B. HARRIS, *Determining bounds on integrals with applications to cataloging problems*, The Annals of Mathematical Statistics **30**, 521–548 (1959).

[10] B. HARRIS, *Determining bounds on expected values of certain functions*, The Annals of Mathematical Statistics **33**, 1454–1457 (1962).

[11] S. KARLIN, & A. NOVIKOFF, *Generalized convex inequalities*, Pacific Journal of Mathematics **13**, 1251–1279 (1963).

[12] S. KARLIN, & L. S. SHAPLEY, *Geometry of moment spaces*, Memoirs **12** of the American Mathematical Society, Providence (1953).

[13] S. KARLIN, & W. J. STUDDEN, *Tchebycheff Systems with Applications in Analysis and Statistics*. Wiley Interscience, New York (1966).

[14] CL. LEFÈVRE, & S. UTEV, *Comparing sums of exchangeable Bernoulli random variables*, Journal of Applied Probability **33**, 285–310 (1996).

[15] T. POPOVICIU, *Sur quelques propriétés des fonctions d'une ou de deux variables réelles*, Mathematica **7**, 1–85 (1933).

[16] T. ROLSKI, *Order relations in the set of probability distribution functions and their applications in queueing theory*, Dissertationes Mathematicae **132**, 5–47 (1976).

[17] M. SHAKED, & J. G. SHANTHIKUMAR, *Stochastic Orders and their Applications*, Academic Press, New York (1994).

[18] D. SOTYAN, *Comparison Methods for Queues and Other Stochastic Models*, Wiley, New York (1983).

[19] J. F. WALHIN, & J. PARIS, *On the use of equispaced discrete distributions*, Astin Bulletin **28**, 241–255 (1998).

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