Journal of Mathematical Inequalities
Volume 2, Number 2 (2008), 215–227

GENERAL WIENER–HOPF EQUATIONS AND NONEXPANSIVE MAPPINGS

MUHAMMAD ASLAM NOOR, S. ZAINAB AND H. YAQOOB

Abstract. In this paper, we show that the general variational inequalities are equivalent to a new class of general Wiener-Hopf equations involving the nonexpansive mappings. Using this equivalence, we suggest and analyze an iterative method for finding the common elements of the solution set of the general variational inequalities and the solution set of the fixed-point of the nonexpansive mapping. We also consider the convergence criteria of the proposed method under some mild conditions. Since the general variational inequalities and the Wiener-Hopf equations include several classes of variational inequalities and Wiener-Hopf equations as special cases, our results continue to hold for these problems. Results obtained in this paper may be viewed as a refinement and improvement of the previously known results.

Mathematics subject classification (2000): 49J40, 90C33.
Key words and phrases: Variational inequalities, fixed point problems, Wiener-Hopf equations, relaxed cocoercive operators, nonexpansive mappings convergence criteria.

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