

SHAFFER–FINK TYPE INEQUALITIES FOR THE ELLIPTIC FUNCTION $\operatorname{sn}(u|k)$

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Abstract. The inequalities of Shafer and Fink, namely,

$$\frac{3x}{2 + \sqrt{1-x^2}} \leq \sin^{-1}(x) \leq \frac{\pi x}{2 + \sqrt{1-x^2}}, \quad x \in [0, 1)$$

are generalized to similar inequalities for the elliptic function $\operatorname{sn}(u|k)$.

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