

## INEQUALITIES FOR THE RATIOS OF CERTAIN BIVARIATE MEANS

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**Abstract.** Inequalities connecting ratios of bivariate homogeneous means whose variables satisfy certain monotonicity conditions are obtained. Derived results include the Stolarsky, Gini, Schwab-Borchardt, and lemniscatic means.

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*Key words and phrases:* Stolarsky means, Gini means, Schwab-Borchardt mean, lemniscatic mean, inequalities.

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