

EXTENSIONS OF THE GENERALIZED WILKER INEQUALITY TO BESSEL FUNCTIONS

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Abstract. In this note our aim is to extend the weighted and exponential generalization of Wilker's inequality and the Sándor-Bencze conjectured inequality to Bessel functions of the first kind. Our main motivation to write this note is a recent publication of Wu and Srivastava, which we wish to complement.

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