

## NORM AND NUMERICAL RADIUS INEQUALITIES FOR A PRODUCT OF TWO LINEAR OPERATORS IN HILBERT SPACES

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*Abstract.* The main aim of the present paper is to establish some norm and numerical radius inequalities for the composite operator  $BA$  under suitable assumptions for the transform  $C_{\alpha,\beta}(T) := (T^* - \bar{\alpha}I)(\beta I - T)$ , where  $\alpha, \beta \in \mathbb{C}$  and  $T \in B(H)$ , of the operators involved.

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