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GRÜSS'S INEQUALITY, ITS PROBABILISTIC INTERPRETATION, AND A SHARPER BOUND

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Abstract. Motivated by statistical and actuarial applications of Grüss's inequality, we argue that the inequality can be sharpened if there is additional information about the mean values of the two functions in Grüss's inequality. In this sense, our research deviates from a large body of literature where Grüss's inequality has been sharpened by imposing more smoothness on the functions.

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