Journal of
Mathematical
Inequalities
Volume 3, Number 1 (2009), 63–77

GENERALIZED HYERS–ULAM–RASSIAS STABILITY OF FUNCTIONAL INEQUALITIES AND FUNCTIONAL EQUATIONS

ZHEN-XIA GAO, HUAI-XIN CAO, WEN-TING ZHENG AND LU XU

Abstract. In this paper, the definitions of the stability of functional inequalities and functional equations are given. Also we prove the generalized Hyers-Ulam-Rassias stability of the following functional inequality and functional equation

$$\|f(x)+f(y)+2f(z)\| \leqslant \left\|2f\left(\frac{x+y}{2}+z\right)\right\|,$$

$$f(x)+f(y)+2f(z)=2f\left(\frac{x+y}{2}+z\right),$$

in the spirit of the Hyers' direct method for approximately additive mappings.

Mathematics subject classification (2000): 39B82.

Keywords and phrases: Functional inequality, functional equation, additivity, stability, generalized Hyers-Ulam-Rassias stability.

REFERENCES

[1] S. M. ULAM, *A Collection of the Mathematical Problems*, Interscience Publ., New York, 1960.
[2] D. H. HYERS, *On the stability of the linear functional equation*, Proc. Natl. Acad. Sci. USA 27 (1941) 222–224.
[3] TH. M. RASSIAS, *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc. 72 (1978) 297–300.
[4] TH. M. RASSIAS, *Problem 16; 2*, in: Report of the 27th International Symp. on Functional Equations, Aequationes Math. 39 (1990) 292–293.
[5] Z. GAJDA, *On stability of additive mappings*, Int. J. Math. Math. Sci. 14 (1991) 431–434.
[6] TH. M. RASSIAS, P. ŠEMRL, *On the behaviour of mappings which do not satisfy Hyers-Ulam stability*, Proc. Amer. Math. Soc. 114 (1992) 989–993.
[7] P. GĂVRUTA, *A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings*, J. Math. Anal. Appl. 184 (1994) (2000) 431–436.
[8] S. JUNG, *On the Hyers-Ulam-Rassias stability of approximately additive mappings*, J. Math. Anal. Appl. 204 (1996) 221–226.
[9] P. CZERWIK, *Functional Equations and Inequalities in Several Variables*, World Scientific, New Jersey, Hong Kong, Singapore and London, 2002.
[10] D. H. HYERS, G. ISAC, TH. M. RASSIAS, *Stability of Functional Equation in Several Variables*, Rirkhäuser, Basel, 1998.
[11] J. M. RASSIAS, *On approximation of approximately linear mappings by linear mappings*, Bull. Sci. Math. 108 (1984) 445–446.
[12] G. ISAC, TH. M. RASSIAS, *Stability of additive mappings: Applications to nonlinear analysis*, Int. J. Math. Math. Sci. 19 (1996) 219–228.
[13] D. H. HYERS, G. ISAC, TH. M. RASSIAS, *On the asymptoticity aspect of Hyers-Ulam stability of mappings*, Proc. Amer. Math. Soc. 126 (1998) 425–420.
[14] C. PARK, *On the stability of the linear mapping in Banach modules*, J. Math. Anal. Appl. 275 (2002) 711–720.

Paper JMI-03-06

[15] C. PARK, *Isomorphisms between unital C^*-algebras*, J. Math. Anal. Appl. 307 (2005) 753–762.
[16] C. PARK, *Hyers-Ulam-Rassias stability of homomorphisms in quasi-Banach algebras*, Bull. Sci. Math. 132 (2008) 87–96.
[17] TH. M. RASSIAS, *The problem of S. M. Ulam for approximately multiplicative mappings*, J. Math. Anal. Appl. 246 (2000) 352–378.
[18] TH. M. RASSIAS, *On the stability of functional equations in Banach spaces*, J. Math. Anal. Appl. 251 (2000) 264–284.
[19] TH. M. RASSIAS, *Functional Equations, Inequalities and Applications*, Kluwer Academic, Dordrecht, Boston and London, 2003.
[20] F. SKOF, *Proprietà localie approssimazione di operatori*, Rend. Sem. Mat. Fis. Milano 53 (1983) 113–129.
[21] J. M. RASSIAS, *On approximation of approximately linear mappings by linear mappings*, J. Funct. Anal. 46 (1982) 126–130.
[22] J. M. RASSIAS, *Solution of a problem of Ulam*, J. Approx. Theory 57 (1989) 268–273.
[23] J. M. RASSIAS, *Complete solution of the multi-dimensional problem of Ulam*, Discuss. Math. 14 (1994) 101–107.
[24] J. M. RASSIAS, M. J. RASSIAS, *On some approximately quadratic mappings being exactly quadratic*, J. Indian Math. Soc. 69 (2002) 155–160.
[25] C. BAAK, D. BOO, TH. M. RASSIAS, *Generalized additive mapping in Banach modules and isomorphisms between C^*-algebras*, J. Math. Anal. Appl. 314 (2006) 150–161.
[26] C. T. NG, *Jensens functional equation on groups*, Aequationes Math. 39 (1990), 85–90.
[27] J. C. PARNAMI, H. L. VASUDEVA, *On Jensens functional equation*, Aequationes Math. 43 (1992), 211–218.
[28] H. HARUKI, TH. M. RASSIAS, *New generalizations of Jensens functional equation*, Proc. Amer. Math. Soc. 123 (1995), 495–503.