

INEQUALITIES AND MONOTONICITY PROPERTIES FOR SOME SPECIAL FUNCTIONS

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Abstract. The monotonicity, convexity, log-convexity and complete monotonicity properties for some special functions are proved, and some inequalities are established.

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Keywords and phrases: Monotonicity, convexity, log-convexity, complete monotonicity, psi function, inequality.

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