

SHARP NORM INEQUALITIES FOR THE TRUNCATED HILBERT TRANSFORM

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Abstract. We split the classical Hilbert Transform H into the sum of two convolution integrals $H^{(\delta)} + R^{(\delta)}$, where the kernel of $H^{(\delta)}$ is supported away from the origin in $\{|t| \geq \delta\}$, while the kernel of $R^{(\delta)}$ is supported near the origin in $\{|t| \leq \delta\}$. We prove that the L^p -norm of $H^{(\delta)}$, known in the literature as the *Truncated Hilbert Transform*, is equal to the norm of H . Namely $\|H^{(\delta)}\|_{p,p} = \cot(\pi/2p)$ for $2 \leq p < +\infty$, and $\|H^{(\delta)}\|_{p,p} = \tan(\pi/2p)$ for $1 < p \leq 2$. We then prove that the L^p -norm of $R^{(\delta)}$ is strictly larger. In particular $\|R^{(\delta)}\|_{2,2} = 1.17897\dots$, a constant related to the Gibbs phenomenon.

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