

SCHUR COMPLEMENTS AND DETERMINANT INEQUALITIES

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Abstract. This paper is focused on the applications of Schur complements to determinant inequalities. It presents a monotonic characterization of Schur complements in the Löwner partial ordering sense such that a new proof of the Hadamard-Fischer-Koteljanski inequality is obtained. Meanwhile, it presents matrix identities and determinant inequalities involving positive semidefinite matrices and extends the Hua Loo-keng determinant inequality by the technique of Schur complements.

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