

SELF-IMPROVEMENT OF THE INEQUALITY BETWEEN ARITHMETIC AND GEOMETRIC MEANS

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Abstract. We show that a simple change of variable allows one to derive from the the AM-GM inequality an improved version of itself. As an application, a refinement of Hölder's inequality for an arbitrary number of functions is obtained.

Mathematics subject classification (2000): 26D15.

Keywords and phrases: self-improvement, arithmetic-geometric inequality, Hölder's inequality.

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