

SCHUR CONVEXITY AND SCHUR-GEOMETRICALLY CONCAVITY OF GENERALIZED EXPONENT MEAN

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Abstract. The monotonicity, the Schur-convexity and the Schur-geometrically convexity with variables (x, y) in \mathbb{R}_{++}^2 for fixed a of the generalized exponent mean $I_a(x, y)$ is proved. Besides, the monotonicity with parameters a in \mathbb{R} for fixed (x, y) of $I_a(x, y)$ is discussed by using the hyperbolic composite function. Furthermore, some new inequalities are obtained.

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