

CAUCHY–RASSIAS STABILITY OF HOMOMORPHISMS ASSOCIATED TO A PEXIDERIZED CAUCHY–JENSEN TYPE FUNCTIONAL EQUATION

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Abstract. We use a fixed point method to prove the Cauchy–Rassias stability of homomorphisms associated to the Pexiderized Cauchy–Jensen type functional equation

$$rf\left(\frac{x+y}{r}\right) + sg\left(\frac{x-y}{s}\right) = 2h(x), \quad r, s \in \mathbb{R} \setminus \{0\}$$

in Banach algebras.

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