ON SUPERQUADRACITY

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Abstract. There are two classes of functions called Superquadratic Functions. In some cases these classes coincide but not always. In this paper this subject is discussed.

Our definition of a superquadratic function is: A function $\varphi : [0.\infty) \to \mathbb{R}$ is superquadratic provided that for all $x \geqslant 0$ there is a constant $C(x) \in \mathbb{R}$ such that

$$\varphi(y) \geqslant \varphi(x) + C(x)(y - x) + \varphi(|y - x|)$$

for all $y \geqslant 0$.

This definition was used in many papers since 2004.

On the other hand, Kominek and Troczka (2006) used W. Smajdor (1987) definition of superquadracity, and in particular, for functions defined on $\mathbb{R}$ their definition is as follows:

The function $\varphi : \mathbb{R} \to \mathbb{R}$ is superquadratic if

$$\varphi(x + y) + \varphi(x - y) \geqslant 2\varphi(x) + 2\varphi(y),$$

is satisfied for all $x, y \in \mathbb{R}$.

After discussing the differences and similarities of these definitions, we show that the class of superquadratic functions as we defined, lead to many applications. Some of these applications we show here.

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