INEQUALITIES IN SUMMABILITY THEORY OF FOURIER SERIES

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Abstract. Some recent results on a general summability method, the so-called θ-summability, are summarized for one-dimensional Fourier series. Natural choices of θ are investigated, i.e., if θ is in Wiener amalgam spaces, Feichtinger's algebra or modulation spaces. Sufficient and necessary conditions are given for the uniform and L_1 norm and a.e. convergence of the θ-means $\sigma_n^\theta f$ to the function f. The maximal operator of the θ-means is investigated and it is proved that it is bounded on L_p spaces and on Hardy spaces.

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