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LOG–CONVEXITY OF COMBINATORIAL SEQUENCES FROM THEIR CONVEXITY

TOMISLAV DOŠLIĆ

Abstract. A sequence $(x_n)_{n\geqslant 0}$ of positive real numbers is log-convex if the inequality $x_n^2 \leqslant x_{n-1}x_{n+1}$ is valid for all $n \geqslant 1$. We show here how the problem of establishing the log-convexity of a given combinatorial sequence can be reduced to examining the ordinary convexity of related sequences. The new method is then used to prove that the sequence of Motzkin numbers is log-convex.

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