

RESOLVENT OPERATOR METHOD FOR GENERAL VARIATIONAL INCLUSIONS

EMAN AL-SHEMAS

Abstract. In this paper, we introduce a new class of variational inclusions involving three operator. Using the resolvent operator technique, we establish the equivalence between the general variational inclusions and the resolvent equations. We use this alternative equivalent formulation to suggest and analyze some iterative methods for solving the general variational inclusions. We also consider the criteria of these iterative methods under suitable conditions. Since the general variational inclusions include the variational inequalities and the related optimization problems as special cases, our results continue to hold for these problems.

Mathematics subject classification (2000): 49J40, 90C33.

Keywords and phrases: variational inclusion, approximation methods, resolvent equations, variational inequalities.

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