FIXED POINTS AND GENERALIZED STABILITY FOR FUNCTIONAL EQUATIONS IN ABSTRACT SPACES

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Abstract. We use a fixed point method, initiated in [V. Radu, Fixed Point Theory 4(2003), No.1, 91-96], to prove the generalized Ulam-Hyers stability of functional equations in single variable for mappings with values in random normed spaces. This result is then used to obtain the stability for Cauchy, quadratic and monomial functional equations.

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