

## THE PROBABILISTIC STABILITY FOR A FUNCTIONAL NONLINEAR EQUATION IN A SINGLE VARIABLE

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*Abstract.* We use the fixed point method to prove the probabilistic Hyers–Ulam and generalized Hyers–Ulam–Rassias stability for the nonlinear equation  $f(x) = \Phi(x, f(\eta(x)))$  where the unknown is a mapping  $f$  from a nonempty set  $S$  to a probabilistic metric space  $(X, F, T_M)$  and  $\Phi : S \times X \rightarrow X$ ,  $\eta : S \rightarrow X$  are two given functions.

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