

## ALMOST EVERYWHERE CONVERGENCE OF A SUBSEQUENCE OF THE NÖRLUND LOGARITHMIC MEANS OF WALSH-KACZMARZ-FOURIER SERIES

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*Abstract.* The main aim of this paper is to prove that the maximal operator of a subsequence of the (one-dimensional) logarithmic means of Walsh-Kaczmarz-Fourier series is of weak type  $(1, 1)$ . Moreover, we prove that the maximal operator of the logarithmic means of quadratical partial sums of double Walsh-Kaczmarz-Fourier series is of weak type  $(1, 1)$ , provided that the supremum in the maximal operator is taken over special indices. The set of Walsh-Kaczmarz polynomials is dense in  $L^1$ , so by the well-known density argument the logarithmic means  $t_{2n}^k(f)$  converge a.e. to  $f$  for all integrable function  $f$ .

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