

ALMOST EVERYWHERE CONVERGENCE OF A SUBSEQUENCE OF THE NÖRLUND LOGARITHMIC MEANS OF WALSH-KACZMARZ-FOURIER SERIES

KÁROLY NAGY

Abstract. The main aim of this paper is to prove that the maximal operator of a subsequence of the (one-dimensional) logarithmic means of Walsh-Kaczmarz-Fourier series is of weak type $(1, 1)$. Moreover, we prove that the maximal operator of the logarithmic means of quadratical partial sums of double Walsh-Kaczmarz-Fourier series is of weak type $(1, 1)$, provided that the supremum in the maximal operator is taken over special indices. The set of Walsh-Kaczmarz polynomials is dense in L^1 , so by the well-known density argument the logarithmic means $t_{2n}^k(f)$ converge a.e. to f for all integrable function f .

Mathematics subject classification (2000): 42C10.

Keywords and phrases: Walsh group, Walsh-Kaczmarz system, double Walsh-Kaczmarz-Fourier series, logarithmic means, a.e. convergence.

REFERENCES

- [1] G.H. AGAEV, N.JA. VILENKIN, G.M. DZHAFARLI, AND A.I. RUBINSTEIN, *Multiplicative systems of functions and harmonic analysis on 0-dimensional groups* (Russian), Izd. ("ELM"), Baku, 1981.
- [2] I. BLAHOTA, G. GÁT, *Norm summability of Nörlund logarithmic means on unbounded Vilenkin groups*, Anal. in Theory and Appl., **24**, 1 (2008), 1–17.
- [3] G. GÁT, *On $(C, 1)$ summability of integrable functions with respect to the Walsh-Kaczmarz system*, Studia Math., **130**, 2 (1998), 135–148.
- [4] G. GÁT, *Investigations of certain operators with respect to the Vilenkin system*, Acta Math. Hung., **61**, 1-2 (1993), 131–149.
- [5] G. GÁT, U. GOGINA, *Uniform and L -convergence of logarithmic means of Walsh-Fourier series*, Acta Math. Sinica, English Series, **22**, 2 (2006), 497–506.
- [6] G. GÁT, U. GOGINA, *Almost everywhere convergence of a subsequence of the logarithmic means of quadratical partial sums of double Walsh-Fourier series*, Publ. Math. Debrecen, **71**, 1-2 (2007), 173–184.
- [7] G. GÁT, R. TOLEDO, *L^p -norm convergence of series in compact, totally disconnected groups*, Analysis Mathematica, **22** (1996), 13–24.
- [8] U. GOGINA, *Almost everywhere convergence of subsequence of logarithmic means of Walsh-Fourier series*, Acta Math. Acad. Paed. Nyíregyháziensis, **21** (2005), 169–175.
- [9] F. MÓRICZ, F. SCHIPP, W.R. WADE, *Cesáro summability of double Walsh-Fourier series*, Trans. Amer. Math. Soc., **329** (1992), 131–140.
- [10] F. MÓRICZ, A. SIDDIQI, *Approximation by Nörlund means of Walsh-Fourier series*, Journal of Approx. Theory, **70**, 3 (1992), 375–389.
- [11] K. NAGY, *On the two-dimensional Marcinkiewicz means with respect to Walsh-Kaczmarz system*, Journal of Approx. Theory, **142** (2006), 138–165.
- [12] F. SCHIPP, W. R. WADE, P. SIMON, AND J. PÁL, *Walsh Series. An Introduction to Dyadic Harmonic Analysis*, Adam Hilger, Bristol-New York, 1990.
- [13] P. SIMON, *Strong convergence of certain means with respect to the Walsh-Fourier series*, Acta Math. Hung., **49** (1987), 425–431.

- [14] P. SIMON, *(C, α) summability of Walsh-Kaczmarz-Fourier series*, J. of Approx. Theory, **127** (2004), 39–60.
- [15] V.A. SKVORTSOV, *On Fourier series with respect to the Walsh-Kaczmarz system*, Analysis Math., **7** (1981), 141–150.
- [16] A. A. ŠNEIDER, *On series with respect to the Walsh functions with monotone coefficients*, Izv. Akad. Nauk SSSR Ser. Math., **12** (1948), 179–192.
- [17] O. SZÁSZ, *On the logarithmic means of rearranged partial sums of a Fourier series*, Bull. Amer. Math. Soc., **48** (1942), 705–711.
- [18] K. YABUTA, *Quasi-Tauberian theorems, applied to the summability of Fourier series by Riesz's logarithmic means*, Tohoku Math. J., II. Ser., **22** (1970), 117–129.