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BOUNDS FOR THE NORMALIZED JENSEN–MERCER FUNCTIONAL

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Abstract. We introduce the normalized Jensen-Mercer functional

$$\mathscr{M}_n(f,\boldsymbol{x},\boldsymbol{p}) = f(a)+f(b)-\sum_{i=1}^{n} p_i f(x_i) - f\left(a+b-\sum_{i=1}^{n} p_i x_i\right)$$

and establish the inequalities of type $M\mathscr{M}_n(f,\boldsymbol{x},\boldsymbol{q}) \geqslant \mathscr{M}_n(f,\boldsymbol{x},\boldsymbol{p}) \geqslant m\mathscr{M}_n(f,\boldsymbol{x},\boldsymbol{q})$, where f is a convex function, $\boldsymbol{x} = (x_1,\dots,x_n)$ and m and M are real numbers satisfying certain conditions. We prove them for the case when $\boldsymbol{p}$ and $\boldsymbol{q}$ are nonnegative n-tuples and when $\boldsymbol{p}$ and $\boldsymbol{q}$ satisfy the conditions for the Jensen-Steffensen inequality. We also give their integral versions in both cases.

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