

## INEQUALITIES OF GRÜSS TYPE INVOLVING THE $p$ -HH-NORMS IN THE CARTESIAN PRODUCT SPACE

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**Abstract.** Inequalities in estimating a type of Čebyšev functional involving the  $p$ -HH-norms are obtained by applying the known results by Grüss, Ostrowski, Čebyšev, and Lupaş. Some of these inequalities are proven to be sharp. In 1998, Dragomir and Fedotov considered a generalised Čebyšev functional, in order to approximate the Riemann-Stieltjes integral. In this paper, some sharp bounds for the generalised Čebyšev functional with convex integrand and monotonically increasing integrator are established as well. An application for the Čebyšev functional involving the  $p$ -HH-norms is also considered; and the bounds are proven to be sharp.

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