

A PROPERTY OF A FUNCTIONAL INCLUSION CONNECTED WITH HYERS–ULAM STABILITY

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Abstract. We prove that a set-valued map $F : X \rightarrow \mathcal{P}_0(Y)$ satisfying the functional inclusion $F(x) \diamond F(y) \subseteq F(x * y)$ admits, in appropriate conditions, a unique selection $f : X \rightarrow Y$ satisfying the functional equation $f(x) \diamond f(y) = f(x * y)$, where $(X, *)$, (Y, \diamond) are square-symmetric grupoids and \diamond is the extension of \diamond to the collection $\mathcal{P}_0(Y)$ of all nonempty parts of Y .

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