

HILBERT INEQUALITY AND GAUSSIAN HYPERGEOMETRIC FUNCTIONS

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Abstract. By using the integral representation of Gaussian hypergeometric function, we obtain Hilbert type inequalities with some fractional kernels and non-conjugate parameters. Such inequalities include the constant factors expressed in terms of hypergeometric functions. Further, we obtain the best possible constants for some general cases, in conjugate case.

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