A NOTE ON CERTAIN MAPS BETWEEN ORDERED FIELDS

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Abstract. We characterize maps f between ordered fields satisfying one of the following five sets of conditions for all x, y:

(A) $f(xy) = f(x)f(y)$, and $f(x+y) \leqslant f(x) + f(y)$,
(B1) $f(xy) = f(x)f(y)$, and $f(x+y) \geqslant f(x) + f(y)$,
(B2) $f(xy) \leqslant f(x)f(y)$, and $f(x+y) = f(x) + f(y)$,
(B3) $f(xy) \geqslant f(x)f(y)$, and $f(x+y) = f(x) + f(y)$.
(C) $f(xy) \geqslant f(x)f(y)$, and $f(x+y) \geqslant f(x) + f(y)$.
Also we pose a problem.

Mathematics subject classification (2000): 12J15, 39B72.
Keywords and phrases: ordered field; homomorphism.

REFERENCES

[1] J. ACZÉL, J. DHOMBRES, *Functional Equations in Several Variables*, Cambridge University Press, 1989.
[2] BOURBAKI, *Algèbre II*, Masson, Paris, 1981.
[3] J. W. S. CASSELS, *Global Fields, in Algebraic Number Theory (eddited by Cassels and Fröhlich)*, Academic Press 1967.
[4] J. DHOMBRES, *Sur les fonctions simultanément suradditives et surmultiplicatives*, C. R. Math. Rep. Acad. Sci. Canada, 5(1983), 207–210.
[5] F. LORENZ, *Algebra II*, Universitext, Springer 2008.
[6] R. S. PIERCE, *Associative Algebras*, Graduate Texts in Mathematics, Springer-Verlag 1982.
[7] M. RĂDULESCU, *On a supra-additive and supra-multiplicative operator in $C(X)$*, Bull. Math. Soc. Sci. Math. R. S. Roumanie 24 (72) (1980), 303–305.
[8] P. VOLKMANN, *Sur les fonctions simultanément suradditives et surmultiplicatives*, Bull. Math. Soc. Sci. Math. R. S. Roumanie 28 (76) (1984), 181–184.
[9] B. L. VAN DER WAERDEN, *Algebra I and II*, Springer-Verlag 1971 and 1967.

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