

ACCURATE APPROXIMATIONS FOR THE RIEMANN–STIELTJES INTEGRAL VIA THEORY OF INEQUALITIES

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Abstract. Accurate approximations for the Riemann-Stieltjes integral by the use of various recent inequalities for the generalised Čebyšev functional introduced in 1998 by Dragomir & Fedotov are surveyed. Applications in deriving sharp inequalities of Grüss' type are also given.

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