A SURVEY OF SOME BOUNDS FOR GAUSS' HYPERGEOMETRIC FUNCTION AND RELATED BIVARIATE MEANS

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Abstract. We give an expository summary of a collection of inequalities involving Gauss' hypergeometric function $_2F_1$ and the closely-related power mean (and certain other bivariate means). Two conjectures involving simultaneous sharp bounds for the hypergeometric function are included. Sharpness for the corresponding zero-balanced case is observed.

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