

## ON SOME BOUNDS OF OSTROWSKI AND ČEBYŠEV TYPE

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*Abstract.* Making use of an identity of Dragomir and Barnett, proved in [13] [published in *J. Indian Math. Soc. (N.S.)*, **66** (1999), No. 1-4, 237-245], some new Ostrowski and Čebyšev type inequalities involving two functions have been developed. Bounds obtained for the new established Ostrowski and Čebyšev type inequalities are of interest and are better than the bounds available in the literature for these type of inequalities.

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