

## AN ITERATIVE ALGORITHM FOR SYSTEM OF MIXED VARIATIONAL-LIKE INEQUALITIES

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*Abstract.* In this paper we consider a system of mixed variational-like inequalities (for short, SMVLI) involving nondifferentiable terms, and its related system of auxiliary problems in the setting of real Hilbert spaces. An existence theorem for the system of auxiliary problems is established. By exploiting this theorem, an iterative algorithm for SMVLI is constructed. We derive the existence of a unique solution of SMVLI and discuss the convergence analysis of the proposed iterative algorithm. Our results represent the generalization, improvement and development of the previously known results in the literature.

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