

## ON ALZER'S INEQUALITY AND ITS GENERALIZED FORMS

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*Abstract.* By using the theory of multiplicatively convex functions, we present some general forms of Alzer's inequality. As consequence, some relevant results in the literature are recovered and an open problem by J. S. Ume [12] is also solved.

*Mathematics subject classification* (2010): 26A51, 26D15.

*Keywords and phrases:* Multiplicative convexity, Alzer's inequality, generalization.

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