EXPONENTIAL CONVEXITY, POSITIVE SEMI–DEFINITE MATRICES AND FUNDAMENTAL INEQUALITIES

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Abstract. The first two chapters of the classical book [6] on inequalities are devoted to fundamental inequalities and positive definiteness. In this paper we obtain results which give connection between fundamental inequalities and positive definiteness using the notion of exponential convexity.

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