

## TRIGONOMETRIC APPROXIMATION IN GENERALIZED LEBESGUE SPACES $L^{p(x)}$

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**Abstract.** The approximation properties of Nörlund  $(N_n)$  and Riesz  $(R_n)$  means of trigonometric Fourier series are investigated in generalized Lebesgue spaces  $L^{p(x)}$ . The deviations  $\|f - N_n(f)\|_{p(x)}$  and  $\|f - R_n(f)\|_{p(x)}$  are estimated by  $n^{-\alpha}$  for  $f \in Lip(\alpha, p(x))$  ( $0 < \alpha \leq 1$ ).

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