

RESTRICTED CURVATURE IN THE MINKOWSKI PLANE

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Abstract. A geometric proof for the following problem is given: Let E be the unit circle in a Minkowski plane. Let C be any continuously differentiable closed curve with length $l(C)$ (measured in Minkowski metric). Assume $|\kappa_e(C, \cdot)| \leq k\kappa_e(E, \cdot)$ and $\kappa_e(E, \cdot)$ denote Euclidean curvatures. Then C can be contained in a similar copy of the unit disk translated and magnified by a factor of

$$\frac{l(C)}{4} - \frac{1}{4k}(l(E) - 4).$$

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