VERY ACCURATE APPROXIMATIONS FOR THE FACTORIAL FUNCTION

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Abstract. We establish the following new Stirling-type approximation formulas for the factorial function

\[ n! \approx \sqrt{2\pi n^n e^{-n}} \left( n + \frac{1}{6} + \frac{1}{72n} - \frac{31}{6480n^2} - \frac{139}{155520n^3} + \frac{9871}{6531840n^4} \right) \]

and

\[ n! \approx \sqrt{2\pi n^n e^{-n}} \left( n^2 + \frac{n}{3} + \frac{1}{18} - \frac{2}{405n} - \frac{31}{9720n^2} \right) \]

Our estimations give much more accurate values for the factorial function than some previously published strong formulas. We also derive new sequences converging to Euler-Mascheroni constant \( \gamma \) very quickly.


Keywords and phrases: Gamma function, factorial function, Stirling formula, psi function, Euler Mascheroni constant, harmonic numbers, inequalities, digamma function.

REFERENCES