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VERY ACCURATE APPROXIMATIONS FOR THE FACTORIAL FUNCTION

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Abstract. We establish the following new Stirling-type approximation formulas for the factorial function

$$n! \approx \sqrt{2\pi} n^n e^{-n} \sqrt{n + \frac{1}{6} + \frac{1}{72n} - \frac{31}{6480n^2} - \frac{139}{155520n^3} + \frac{9871}{6531840n^4}}$$

and

$$n! \approx \sqrt{2\pi} n^n e^{-n} \sqrt[4]{n^2 + \frac{n}{3} + \frac{1}{18} - \frac{2}{405n} - \frac{31}{9720n^2}}.$$

Our estimations give much more accurate values for the factorial function than some previously published strong formulas. We also derive new sequences converging to Euler-Mascheroni constant γ very quickly.

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