Journal of
Mathematical
Inequalities
Volume 4, Number 3 (2010), 391–398

FURTHER EXTENSION OF FURUTA INEQUALITY

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Abstract. If $A_{2n} \geqslant A_{2n-1} \geqslant \cdots \geqslant A_2 \geqslant A_1 \geqslant B \geqslant 0$, with $A_1 > 0$, $t_1, t_2, \cdots, t_{n-1}$, $t_n \in [0,1]$ and $p_1, p_2, \cdots, p_{2n-1}, p_{2n} \geqslant 1$ for a natural number n. Then the following inequality holds for $r \geqslant t_n$

$$\begin{aligned} {A_{2n}}^{1-t_n+r} \geqslant \{&{A_{2n}}^{\frac{r}{2}}[{A_{2n-1}}^{\frac{-t_n}{2}}\{{A_{2(n-1)}}^{\frac{t_{n-1}}{2}} \cdots {A_4}^{\frac{t_2}{2}}[{A_3}^{\frac{-t_2}{2}}\{{A_2}^{\frac{t_1}{2}}({A_1}^{\frac{-t_1}{2}} B^{p_1} {A_1}^{\frac{-t_1}{2}})^{p_2} \\ &{A_2}^{\frac{t_1}{2}}\}^{p_3} {A_3}^{\frac{-t_2}{2}}]^{p_4} {A_4}^{\frac{t_2}{2}} \cdots {A_{2(n-1)}}^{\frac{t_{n-1}}{2}}\}^{p_{2n-1}} {A_{2n-1}}^{\frac{-t_n}{2}}]^{p_{2n}} {A_{2n}}^{\frac{r}{2}}\}^{\frac{1-t_n+r}{\eth[2n]-t_n+r}}, \end{aligned}$$

where $\eth[2n] = \{\cdots[\{[(p_1-t_1)p_2+t_1]p_3-t_2\}p_4+t_2]p_5-\cdots-t_n\}p_{2n}+t_n$.

Mathematics subject classification (2010): 47A63.

Keywords and phrases: Furuta inequality, generalized Furuta inequality, order preserving operator inequality.

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