

AN AREA INEQUALITY FOR ELLIPSES INSCRIBED IN QUADRILATERALS

ALAN HORWITZ

Abstract. If E is any ellipse inscribed in a convex quadrilateral, \mathcal{D} , then we prove that $\frac{\text{Area}(E)}{\text{Area}(\mathcal{D})} \leq \frac{\pi}{4}$, and equality holds if and only if \mathcal{D} is a parallelogram and E is tangent to the sides of \mathcal{D} at the midpoints. We also prove that the foci of the unique ellipse of maximal area inscribed in a parallelogram, \mathcal{D} , lie on the orthogonal least squares line for the vertices of \mathcal{D} . This does not hold in general for convex quadrilaterals.

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