

## AN ANALYTIC SOLUTION FOR SOME SEPARABLE CONVEX QUADRATIC PROGRAMMING PROBLEMS WITH EQUALITY AND INEQUALITY CONSTRAINTS

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*Abstract.* In this paper we provide a complete analytic solution to a particular separable convex quadratic programming problem with bound and equality constraints. This study constitutes the generalization of prior papers in which additional simplifications were considered. We present an algorithm that leads to determination of the analytic optimal solution. We demonstrate that our algorithm is able to deal with large-scale QP problems of this type. Finally, we present an very important application: the classical problem of economic dispatch.

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