

AN ANALYTIC SOLUTION FOR SOME SEPARABLE CONVEX QUADRATIC PROGRAMMING PROBLEMS WITH EQUALITY AND INEQUALITY CONSTRAINTS

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Abstract. In this paper we provide a complete analytic solution to a particular separable convex quadratic programming problem with bound and equality constraints. This study constitutes the generalization of prior papers in which additional simplifications were considered. We present an algorithm that leads to determination of the analytic optimal solution. We demonstrate that our algorithm is able to deal with large-scale QP problems of this type. Finally, we present an very important application: the classical problem of economic dispatch.

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REFERENCES

- [1] C. AUDET, P. HANSEN, S. LE DIGABEL, *Exact solution of three nonconvex quadratic programming problems*, In *Frontiers in Global Optimization*, Volume 20 of *Nonconvex Optimization and Applications*, Kluwer Acad. Publ., Dordrecht (2004), 25–45.
- [2] L. BAYÓN, J. M. GRAU, P. M. SUÁREZ, *A New Formulation of the Equivalent Thermal in Optimization of Hydrothermal Systems*, *Math. Probl. Eng.*, **8**, 3 (2002), 181–196.
- [3] L. BAYÓN, J. M. GRAU, M.M. RUIZ, P. M. SUÁREZ, *The First Weierstrass-Erdmann Condition in Variational Problems involving Differential Inclusions*, *Math. Inequal. Appl.*, **7**, 3 (2004), 457–469.
- [4] L. BAYÓN, J. M. GRAU, M.M. RUIZ, P. M. SUÁREZ, *New developments on equivalent thermal in hydrothermal optimization: an algorithm of approximation*, *J. Comput. Appl. Math.*, **175**, 1 (2005), 63–75.
- [5] R. CAMBINI, *Soluzione ottima esplicita di un particolare problema di programmazione quadratica*, *Decisions in Economics and Finance*, Springer, Milan, **15**, 2 (1992), 73–86.
- [6] S. COSARES, D.S. HOCHBAUM, *Strongly polynomial algorithms for the quadratic transportation problem with a fixed number of sources*, *Math. Oper. Res.*, **19**, 1 (1994), 94–111.
- [7] K. DAHIYA, S.K. SUNEJA, V. VERMA, *Convex programming with single separable constraint and bounded variables*, *Computational Optimization and Applications. An International Journal*, **36**, 1 (2007), 67–82.
- [8] Z. DOSTAL, *Inexact semimonotonic augmented Lagrangians with optimal feasibility convergence for convex bound and equality constrained quadratic programming*, *SIAM J. Numer. Anal.*, **43**, 1 (2005), 96–115.
- [9] Z. DOSTAL, *An optimal algorithm for bound and equality constrained quadratic programming problems with bounded spectrum*, *Computing*, **78**, 4 (2006), 311–328.
- [10] C. J. GOH AND X. Q. YANG, *Analytic efficient solution set for multi-criteria quadratic programs*, *European J. Oper. Res.*, **92**, 1 (1996), 166–181.
- [11] N.I.M. GOULD, PH.L. TOINT, *A Quadratic Programming Bibliography*, http://www.optimization-online.org/DB_HTML/2001/02/285.html, 2001.
- [12] N.I.M. GOULD, PH.L. TOINT, *A Quadratic Programming Page*, <http://www.numerical.rl.ac.uk/qp/qp.html>.

- [13] W.W. HAGER, S. PARK, *The gradient projection method with exact line search*, Journal of Global Optimization, **30**, 1 (2004), 103–118.
- [14] S. KIM, M. KOJIMA, *Exact Solutions of Some Nonconvex Quadratic Optimization Problems via SDP and SOCP Relaxations*, Comput. Optim. Appl., **26**, 2 (2003), 143–154.
- [15] P. A. LOTITO, *Issues in the implementation of the DSD algorithm for the traffic assignment problem*, European Journal of Operational Research, **175**, 3 (2006), 1577–1587.
- [16] N. MEGIDDO, A. TAMIR, *Linear time algorithms for some separable quadratic programming problems*, Oper. Res. Lett., **13** (1993), 203–211.
- [17] H.D. MITTELMANN, *Decision Tree for Optimization Software*, <http://plato.asu.edu/guide.html>.
- [18] S.M. STEFANOV, *Convex Separable Minimization Subject to Bounded Variables*, Computational Optimization and Applications, **18**, 1 (2001), 27–48.
- [19] S.M. STEFANOV, *Separable Programming. Theory and Methods*, Kluwer Academic Publishers: Dordrecht-Boston-London, 2001.
- [20] S.M. STEFANOV, *Convex quadratic minimization subject to a linear constraint and box constraints*, Appl. Math. Res. eXpress, **1** (2004), 17–42.
- [21] S.M. STEFANOV, *Polynomial algorithms for projecting a point onto a region defined by a linear constraint and box constraints in \mathbb{R}^n* , J. Appl. Math., **2004**, 5 (2004), 409–431.