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SUMS OF REAL PARTS OF EIGENVALUES OF PERTURBED MATRICES

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Abstract. Let A be $\tilde{A}$ be $n\times n$ matrices, whose eigenvalues are λ_k and $\tilde{\lambda}_k$, respectively. Assuming that A is Hermitian, we prove the inequality

$$[\sum_{k=1}^{n} |\text{ Re } \tilde{\lambda}_k - \lambda_k|^p]^{1/p} \leqslant N_p(E_R) + \tilde{b}_p N_p(E_I) \ \ (2 \leqslant p < \infty)$$

where $N_p(A)$ is the Schatten-von Neumann norm of A, $E = \tilde{A} - A$, $E_R = (E+E^*)/2$, $E_I = (E-E^*)/2i$, and $\tilde{b}_p \leqslant pe^{1/3}$. That inequality is generalized then to the Schatten-von Neumann operators.

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