SHARP BOUNDS FOR SEIFFERT MEANS
IN TERMS OF LEHMER MEANS

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Abstract. In this paper, we establish two sharp inequalities as follows: $P(a,b) > L_{-\frac{1}{6}}(a,b)$ and $T(a,b) < L_{\frac{1}{3}}(a,b)$ for all $a,b > 0$ with $a \neq b$. Here, $L_r(a,b)$, $P(a,b)$ and $T(a,b)$ are the Lehmer, first and second Seiffert means of a and b, respectively.

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