Journal of Mathematical Inequalities
Volume 5, Number 1 (2011), 1–11

SOME PROPERTIES FOR A CLASS OF SYMMETRIC FUNCTIONS AND APPLICATIONS

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Abstract. For $x=(x_1,x_2,\cdots,x_n)\in\mathbb{R}_+^n$, the symmetric function $F_n(x,r)$ is defined by

$$F_n(x,r)=F_n(x_1,x_2,\cdots,x_n;r)=\prod_{1\leqslant i_1<i_2\cdots<i_r\leqslant n}\frac{\sum_{j=1}^r x_{i_j}}{\sum_{j=1}^r(1+x_{i_j})},$$

where $r=1,2,\cdots,n$ and $i_1,i_2,\cdots,i_n$ are positive integers. In this article, the Schur convexity, Schur harmonic convexity and Schur multiplicative convexity of $F_n(x,r)$ are discussed. As applications, some inequalities are established by use of the theory of majorization.

Mathematics subject classification (2010): 05E05, 26B25.

Keywords and phrases: Symmetric function, Schur convex, Schur harmonic convex, Schur multiplicatively convex, theory of majorization.

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Paper JMI-05-01