

## GENERALIZATIONS OF CONVERSE JENSEN'S INEQUALITY AND RELATED RESULTS

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*Abstract.* In this paper we prove generalizations of Converse Jensen's inequality for convex functions defined on convex hulls. As consequences we get generalizations of the Hermite-Hadamard inequality for convex functions defined on  $k$ -simplices in  $\mathbb{R}^k$ . We also present some related results which generalize results in [8].

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