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THE STEIN–WEISS TYPE INEQUALITIES FOR THE B–RIESZ POTENTIALS

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Abstract. We establish two inequalities of Stein-Weiss type for the Riesz potential operator $I_{\alpha,\gamma}$ ($B-$Riesz potential operator) generated by the Laplace-Bessel differential operator Δ_B in the weighted Lebesgue spaces $L_{p,|x|^\beta,\gamma}$. We obtain necessary and sufficient conditions on the parameters for the boundedness of $I_{\alpha,\gamma}$ from the spaces $L_{p,|x|^\beta,\gamma}$ to $L_{q,|x|^{-\lambda},\gamma}$, and from the spaces $L_{1,|x|^\beta,\gamma}$ to the weak spaces $WL_{q,|x|^{-\lambda},\gamma}$. In the limiting case $p=Q/\alpha$ we prove that the modified $B-$Riesz potential operator $\widetilde{I}_{\alpha,\gamma}$ is bounded from the spaces $L_{p,|x|^\beta,\gamma}$ to the weighted $B-BMO$ spaces $BMO_{|x|^{-\lambda},\gamma}$.

As applications, we get the boundedness of $I_{\alpha,\gamma}$ from the weighted B-Besov spaces $B^s_{p\theta,|x|^\beta,\gamma}$ to the spaces $B^s_{q\theta,|x|^{-\lambda},\gamma}$. Furthermore, we prove two Sobolev embedding theorems on weighted Lebesgue $L_{p,|x|^\beta,\gamma}$ and weighted B-Besov spaces $B^s_{p\theta,|x|^\beta,\gamma}$ by using the fundamental solution of the B-elliptic equation $\Delta_B^{\alpha/2}$.

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