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ON CERTAIN SEQUENCES DERIVED FROM GENERALIZED EULER–MASCHERONI CONSTANTS

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Abstract. Let $0 < \alpha < 1$, and let

$$C_\alpha := \lim_{n\to\infty}\left(1+\frac{1}{2^\alpha}+\cdots+\frac{1}{n^\alpha}-\frac{n^{1-\alpha}}{1-\alpha}\right).$$

It is proved that there exists a unique sequence (ω_n) such that

$$1+\frac{1}{2^\alpha}+\cdots+\frac{1}{n^\alpha}=C_\alpha+\frac{(n+\omega_n)^{1-\alpha}}{1-\alpha}.$$

Moreover, the sequence (ω_n) is decreasing and satisfies $\frac{1}{2}\leqslant\omega_n\leqslant\frac{1}{4}\left[1+\left(1+\frac{1}{n}\right)^\alpha\right]$, whence $\lim_{n\to\infty}\omega_n=\frac{1}{2}$. This is only a special case of the more general results established in this paper. These results concern some sequences derived from generalized Euler–Mascheroni constants involving convex functions and complement similar ones obtained by V. Timofte [Integral estimates for convergent positive series. *J. Math. Anal. Appl.* **303** (2005), 90–102].

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REFERENCES

[1] W. W. BRECKNER AND T. TRIF, *Convex Functions and Related Functional Equations. Selected Topics*, Cluj University Press, Cluj-Napoca, 2008.
[2] D. W. DETEMPLE, *A quicker convergence to Euler's constant*, Amer. Math. Monthly, **100** (1993), 468–470.
[3] G. H. HARDY, J. E. LITTLEWOOD AND G. PÓLYA, *Inequalities*, Cambridge University Press, 1934.
[4] C. P. NICULESCU AND L.-E. PERSSON, *Convex Functions and Their Applications. A Contemporary Approach*, Springer-Verlag, New York, 2006.
[5] A. W. ROBERTS AND D. E. VARBERG, *Convex Functions*, Academic Press, New York, London, 1973.
[6] J. SÁNDOR, *On generalized Euler constants and Schlömilch–Lemonnier type inequalities*, J. Math. Anal. Appl., **328** (2007), 1336–1342.
[7] V. TIMOFTE, *Integral estimates for convergent positive series*, J. Math. Anal. Appl., **303** (2005), 90–102.
[8] L. TOTH, *Problem E3432*, Amer. Math. Monthly, **98** (1991), 264.
[9] L. TOTH, *Problem E3432 (Solution)*, Amer. Math. Monthly, **99** (1992), 684–685.

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