

A SELF-ADAPTIVE PROJECTION METHOD FOR A CLASS OF VARIANT VARIATIONAL INEQUALITIES

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Abstract. In this paper, we consider the general variant variational inequality of the type: Find a vector $u^* \in \mathbb{R}^n$, such that

$$Q(u^*) \in \Omega, \quad \langle v - Q(u^*), Tu^* \rangle \geq 0, \quad \forall v \in \Omega,$$

where T, Q are operators. We suggest and analyze a very simple self-adaptive iterative method for solving this class of general variational inequalities. Under certain conditions, the global convergence of the proposed method is proved. An example is given to illustrate the efficiency and implementation of the proposed method. Preliminary numerical results show that the proposed method is applicable.

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