

STRONG CONVERGENCE THEOREMS OF MODIFIED MANN ITERATIVE PROCESS FOR NONEXPANSIVE MAPPINGS IN HILBERT SPACES

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Abstract. The purpose of this article is to modify normal Mann's iterative process to have strong convergence for nonexpansive mappings in the formwork of Hilbert spaces. We prove the strong convergence of the proposed iterative algorithm to the fixed point of nonexpansive mappings which is the unique solution of a variational inequality, which is also the optimality condition for a minimization problem.

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